Proper Circular Arc Graphs and Circular Robinson Spaces

Introduction

A proper interval graphs is the intersection graph of a family of intervals on the line with no interval properly containing any other. A proper circular arc (PCA) graph is the intersection model of a set of arcs on the circle with no arc properly containing any other.

A similarity space is a set X along with a similarity measure s where $s(a, b) = s(b, a) \geq 0$ is the similarity between $a, b \in X$. A similarity space (X, s) is called a circular Robinson space if there is a circular ordering consistent with the similarity measure.

I will present an overview of an algorithm for recognizing PCA graphs that builds on an algorithm of Corneil [1] and a result stating that a similarity space (X, s) with the property that $s(a, b) = \{0, 1\}$ for all $a, b \in X$ is a circular Robinson space if and only if its similarity matrix is the adjacency matrix of a PCA graph. I will discuss how this result can be applied to the problem of circular robinson space recognition.

Below are some definitions and a more detailed description of the presentation.

Definitions

Proper Interval Model
A proper interval model of a graph G is an assignment of each vertex v of

G to pairs of endpoints (v_l, v_r) and a linear ordering of those endpoints so that v is adjacent to w if and only if $v_l < w_l < v_r$ or $w_l < v_l < w_r$ and there are no vertices v, w with $v_l < w_l < w_r < v_r$. Notice that in a proper interval model $v_l < w_l \longleftrightarrow v_r < w_r$.

Proper Interval Graph
A proper interval graph is a graph that has a proper interval model

Proper Circular Arc Model
A proper circular arc model of a graph G is an assignment of each vertex $v \in V(G)$ to a pair (v_l, v_r) called left and right endpoints and a circular order of the endpoints such that v is adjacent to w if and only if $v_l < w_l < v_r$ or $w_l < v_l < v_r$ and we never have $v_l < w_l < w_r < v_r$ for any v, w in G. G is a proper circular arc (PCA) graph if it has a PCA model.

Proper Circular Arc Graph
A proper circular arc graph is a graph admitting a proper circular arc model

Similarity Space
A similarity space is a set X with a similarity measure s where $s(a, b) = s(b, a) \geq 0$ is the similarity between elements $a \neq b$ in X

Binary Similarity Space
A binary similarity space is a similarity space (X, s) where $s(a, b) = \{0, 1\}$ for all a, b in X and $s(a, a) = 1$ for all $a \in X$ and $s(a, a) = 1$ for all $a \in X$

Circular Robinson Space
A similarity space (X, s) is a circular robinson space if there is a circular order $<$ on the elements of X such that the following condition is satisfied:

$$x < y < z < t \Longrightarrow s(x, z) \leq max\{min\{s(x, y), s(y, z)\}, min\{s(x, t), s(t, z)\}\}$$

Presentation

I will present an overview breadth first search based algorithm for recognizing PCA graphs that builds on an algorithm from [1]. I will also present the following:

A binary similarity space $S = (X, s)$ is a circular Robinson space if and only if the similarity matrix of S is the augmented adjacency matrix of some PCA graph G

I will discuss the consequences of this relationship between PCA graphs and circular Robinson spaces. It allows us to characterize non-binary circular Robinson spaces in terms of PCA graphs and could possibly be applied to the problem of recognizing circular Robinson spaces.

References

[1] Derek G Corneil, Hiryoung Kim, Sridhar Natarajan, Stephan Olariu, and Alan P Sprague. Simple linear time recognition of unit interval graphs. *Information processing letters*, 55(2):99–104, 1995.