

Abstract for Maximum Nim with a Pass using Fractional Restrictions

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Combinatorial game theory studies games that are deterministic, have two players, each player has perfect information about the game, and the game will end eventually. In this field of mathematics, there are partial games and impartial games. Partial games are those where the move-set for one player is different from that of the other player, and impartial games are those where the move-sets are the same. Nim is a classic impartial game in the field of combinatorial game theory. We set out to study one of its variants, namely, maximum Nim. In this variant, a player may at most take a fraction of a pile of their choosing rounded up, and at least one stone (unless they use the pass move). In addition to having this maximum restriction, we also consider a pass move that can be played only once in a game, and we provide a relationship between maximum Nim with a pass and without a pass. In the analysis below, we first set out to find the winning positions for these games with certain fractions as maximum restrictions, namely those of the form $\frac{l-1}{l}$ (where l is a positive integer) and then for the rest. The winning positions can be determined via integer sequences and recurrence relations. Beyond the winning positions is the Sprague-Grundy integer function, which assigns a positive integer to a move. Once the winning positions have been determined via the aforementioned methods, the Sprague-Grundy value for any pile size can be determined similarly. Finally, we study the game with multiple piles of stones, relying on the important theorem that the Sprague-Grundy value of a game is the nim-sum of the individual piles involved, with and without a pass.