The exact synthesis problem is an important problem in the theory of quantum computing, where one breaks down a given operator into simpler ones to maximize efficiency and reduce cost. Viewing operators as matrices, the problem becomes a matrix factorization problem. In this talk, I will introduce and compare two methods for the exact synthesis of 2x2 unitary matrices, focusing on the case of matrices with entries in the ring $\mathbb{Z}[1/2, \omega]$, where omega is an eighth root of unity. The first method works by considering the columns of the matrix one by one, while the second method considers the matrix globally by interpreting it as a quaternion. Afterwards, I will discuss possible extensions of both methods, and some of the difficulties one might face in generalizing them.